GENERAL APTITUDE

Q. No. 1 - 5 Carry One Mark Each

1. If IMHO = JNIP; IDK=JEL; and SO = TP, then IDC= ________.

(A) JDE (B) JDC (C) JCD (D) JED

Key: (D)

Sol: IMHO = JNIP IDK = JEL SO = TP

$$\begin{array}{cccc} I & M & H & O \\ +1 & +1 & +1 & +1 \\ J & N & I & P \end{array} \quad \begin{array}{ccc} I & D & K \\ +1 & +1 & +1 \\ J & E & L \end{array} \quad \begin{array}{cc} S & O \\ +1 & +1 \\ T & P \end{array} \quad \begin{array}{ccc} I & D & C \\ \downarrow^{+1} & \downarrow^{+1} & \downarrow^{+1} \\ J & E & D \end{array}$$

2. Once the team of analysts identify the problem, we ______ in a better position to comment on the issue.

Which one of the following choices CANNOT fill the given blank?

(A) might be (B) were to be (C) are going to be (D) will be

Key: (B)

3. The product of three integers X, Y and Z is 192. Z is equal to 4 and P is equal to the average of X and Y. What is the minimum possible value of P?

(A) 7 (B) 6 (C) 9.5 (D) 8

Key: (A)

Sol: Given X, Y, Z = 192

$Z = 4$

$$XY = \frac{192}{4} = 48$$

Possible values of X & Y for XY=48 are

$X = 48$ or 1, $Y = 1$ or 48

$X = 16$ or 3, $Y = 3$ or 16

$X = 12$ or 4, $Y = 4$ or 12

$X = 8$ or 6, $Y = 6$ or 8

$$\text{min values of } P = \frac{X+Y}{2} = \frac{6+8}{2} = 7$$

4. A final examination is the _____ of a series of evaluations that a student has to go through.

(A) insinuation (B) culmination (C) desperation (D) consultation

Key: (B)

5. Are there enough seats here? There are _____ people here than I expected.

(B) least (C) many (A) most (D) more

Key: (D)

Q. No. 6 - 10 Carry Two Marks Each

6. X is an online media provider. By offering unlimited and exclusive online content at attractive prices for a loyalty membership, X is almost forcing its customers towards its loyalty membership. If its loyalty membership continues to grow at its current rate, within the next eight years more households will be watching X than cable television.

Which one of the following statements can be inferred from the above paragraph?

(A) The X is cancelling accounts of non-members

(B) Non-members prefer to watch cable television

(C) Most households that subscribe to X's loyalty membership discontinue watching cable television

(D) Cable television operators don't subscribe to X's loyalty membership

Key: (C)

7. Two pipes P and Q can fill a tank in 6 hours and 9 hours respectively, while a third pipe R can empty the tank in 12 hours. Initially, P and R are open for 4 hours. Then P is closed and Q is opened. After 6 more hours R is closed. The total time taken to fill the tank (in hours) is ____.

(A) 16.50 (B) 14.50 (C) 13.50 (D) 15.50

Key: (B)

Sol: P & Q can fill tank in 6 hours and 9 hours respectively

In 1 hour

P along can fill = $\frac{1}{6}$

Q along can fill = $\frac{1}{9}$

R along can fill = $\frac{1}{12}$

In first four hours P and R are opened

$$4\times P-4R=4\times\frac{1}{6}-4\times\frac{1}{12}=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

After 4 hours P is closed and Q and R opened for 6 more hours

$$6\times Q-6R=6\times\frac{1}{9}-6\times\frac{1}{12}=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$$

In 10 hours tank filled $= \frac{1}{3}+\frac{1}{6}=\frac{1}{2}$

Remaining $= \frac{1}{2}$ tank

Q will take to fill remain half tank $= \frac{9}{2}$hour $=4.5$ hours

Total time taken $= 4+6+4.5=14.5$ hrs.

8. Mola is a digital platform for taxis in a city. It offers three types of rides – Pool, Mini and Prime. The table below presents the number of rides for the past four months. The platform earns one US dollar per ride. What is the percentage share of revenue contributed by Prime to the total revenues of Mola, for the entire duration?

	Month			
Type	**January**	**February**	**March**	**April**
Pool	170	320	215	190
Mini	110	220	180	70
Prime	75	180	120	90

(A) 16.24 (B) 23.97 (C) 25.86 (D) 38.74

Key: (B)

Sol:

	Month					
Type	Jan	Feb	Mar	Apr	Total no. of rides	Revenu 1$ per ride
Pool	170	320	215	190	895	895$
Mini	110	220	180	70	580	580$
Prime	75	180	120	90	465	465$
					Total revenue	1940$

Percentage Share of revenue contributed by prime

$$= \frac{\text{Revenue of prime}}{\text{Total revenue}} \times 100 = \frac{465}{1940} \times 100 = 23.97\%$$

9. Fiscal deficit was 4% of the GDP in 2015 and that increased to 5% in 2016. If the GDP increased by 10% from 2015 to 2016, the percentage increase in the actual fiscal deficit is

(A) 37.50 (B) 25.00 (C) 35.70 (D) 10.00

Key: (A)

Sol: Let us take, In 2015, Fiscal deficit =X, GDP = y

Given

$\text{in } 2015\ \ x = 4\% \text{ of } y = 0.04y$

$\text{in } 2016\ \ y' = \text{increases by } 10\% = 1.10y$

$x' = \text{increases to } 5\% \text{ of } y' = 0.05 \times y'$

$x' = 0.05 \times 1.1y = 0.055y$

$$\text{Percentage increase} = \frac{0.055y - 0.04y}{0.04y} \times 100 = 37.5\%$$

10. While teaching a creative writing class in India, I was surprised at receiving stories from the students that were all set in distant places: in the American West with cowboys and in Manhattan penthouses with clinking ice cubes. This was, till an eminent Caribbean writer gave the writers in the once-colonised countries the confidence to see the shabby lives around them as worthy of being "told".

The writer of this passage is surprised by the creative writing assignments of his students because ______.

(A) None of the students had written stories set in India

(B) Some of the students had written about ice cubes and cowboys

(C) Some of the students had written stories set in foreign places

(D) None of the students had written about ice cubes and cowboys

Key: (A)

MECHANICAL ENGINEERING

Q. No. 1 to 25 Carry One Mark Each

1. If x is the mean of data 3, x, 2 and 4, then the mode is

Key: (3)

Sol: Given data values are 3, x, 2 and 4

$$\therefore \text{ Mean } = \frac{3 + x + 2 + 4}{4} \Rightarrow x = \frac{9 + x}{4} [\because \text{Mean} = x]$$

$\Rightarrow 4x = 9 + x \Rightarrow 3x = 9$

$\Rightarrow x = 3$

$\therefore$ Data value sare3, 3, 2, 4

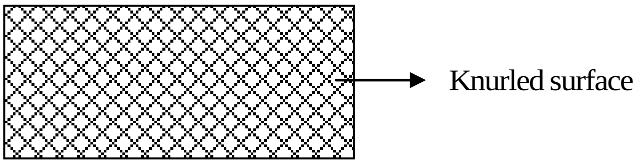
Mode = 3 [most frequently repeated observation]

2. The cold forming process in which a hardened tool is pressed against a workpiece (when there is relative motion between the tool and the workpiece) to produce a roughened surface with a regular pattern is

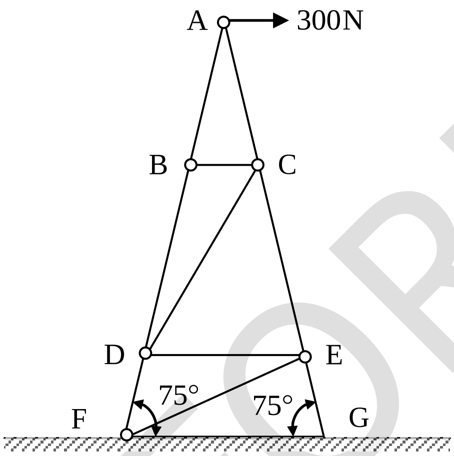
(A) Chamfering (B) Roll forming (C) Knurling (D) Strip rolling

Key: (C)

Sol:

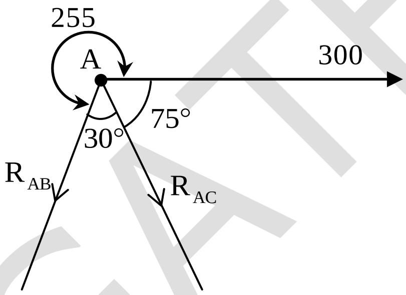


3. The figure shows an idealized plane truss. If a horizontal force of 300N is applied at point A, then the magnitude of the force produced in member CD is ______N.



Key: (0)

Sol: Joint A



$$\frac{300}{\sin 30°} = \frac{R_{AC}}{\sin 255°} = \frac{R_{AB}}{\sin 75°}$$

$$R_{AC} = \frac{300 \sin 255}{\sin 30°} = -579.55\ N(C)$$

$$R_{AB} = 579.55N(T)$$

From the figure AB and BD are collinear. So, $R_{BC} = 0$ and AC and CE are collinear then R_{CD}=0.

4. The fluidity of molten metal of cast alloys (without any addition of fluxes) increases with increase in

(A) viscosity
(B) degree of superheat
(C) surface tension
(D) freezing range

Key: (B)

5. Consider a linear elastic rectangular thin sheet of metal, subjected to uniform uniaxial tensile stress of 100 MPa along the length direction. Assume plane stress condition in the plane normal to the thickness. The Young's modulus E = 200 MPa and Poisson's ratio $v = 0.3$ are given. The principal strains in the plane of the sheet are

(A) (0.35, –0.15)
(B) (0.5, –0.5)
(C) (0.5, 0.0)
(D) (0.5, –0.15)

Key: (D)

Sol: $\sigma_1 = 100\text{MPa},\ \mu = 0.3,\ E = 200\text{MPa},\ \sigma_2 = 0$

$$\in_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \Rightarrow \in_1 = \frac{100}{200} = 0.5$$

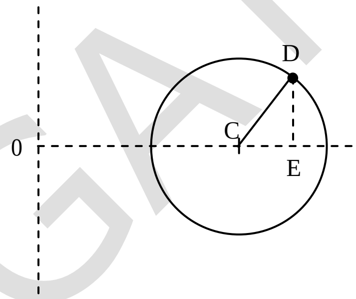
$$\in_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \Rightarrow \in_2 = \frac{-0.3\times 100}{200} = -0.15$$

$$(\in_1, \in_2) = (0.5, -0.15) \therefore \text{option}(D)$$

6. The state of stress at a point in a component is represented by a Mohr's circle of radius 100 MPa centered at 200 MPa on the normal stress axis. On a plane passing through the same point, the normal stress is 260 MPa. The magnitude of the shear stress on the same plane at the same point is ____ MPa.

Key: (80)

Sol:



$$R = 100\text{MPa},\ C = 200\text{MPa},\ \sigma_\theta = 260\text{MPa},\ \tau_\theta = ?$$

$$OC = 200,\ CD = 100,\ OE = 260,\ DE = ?$$

$$DE = \sqrt{CD^2 - CE^2} = \sqrt{100^2 - (260-200)^2} = \sqrt{100^2 - 60^2} = 80\text{MPa}$$

7. A wire of circular cross-section of diameter 1.0 mm is bent into a circular are of radius 1.0 m by application of pure bending moments at its ends. The Young's modulus of the material of the wire is 100 GPa. The maximum tensile stress developed in the wire is ____ MPa.

Key: (50)

Sol: $d = 1mm,\ \ R = 1.0mts,\ \ E = 100GPa,\ \sigma = ?$

$$\sigma = \frac{E}{R}y \qquad \left(y = \frac{1}{2} = 0.5mm\right)$$

$$\sigma = \frac{2100\times10^3}{1000}\times\left(\frac{1}{2}\right) = 50MPa$$

8. A two-dimensional incompressible frictionless flow field is given by $\vec{u} = x\hat{i} - y\hat{j}$. If ρ is the density of the fluid, the expression for pressure gradient vector at any point in the flow field is given as

(A) $\rho\left(x\hat{i} - y\hat{j}\right)$ (B) $-\rho\left(x^2\hat{i} + y^2\hat{j}\right)$ (C) $\rho\left(x\hat{i} + y\hat{j}\right)$ (D) $-\rho\left(x\hat{i} + y\hat{j}\right)$

Key: (D)

Sol: Given that, flow field $\vec{V} = x\hat{i} - y\hat{i}$

$$\vec{V} = u\hat{i} + v\hat{j}$$

Euler's equation of motion in 'x' and 'y' directions are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{du}{dy} + w\frac{\partial u}{\partial z} = x_x - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = x_y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

Where u = x, v = -y, w = 0 x_x, x_y are body forces in 'x' and 'y' directions and they are equal to zero since there is no extra forces acting on fluid.

$\frac{\partial p}{\partial x}$ = pressure gradient in x-directoin; $\frac{\partial p}{\partial y}$ = pressure gradient in y-directoin

$$\therefore\ \frac{\partial}{\partial t}(x) + (x)\frac{\partial}{\partial x}(x) + (-y)\left(\frac{\partial}{\partial y}(x)\right) + 0 = 0 - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$0 + x + 0 = \frac{-1}{\rho}\frac{\partial p}{\partial x} \Rightarrow \frac{\partial p}{\partial x} = -\rho x$$

$$\therefore\ \frac{\partial}{\partial t}(-y) + x\frac{\partial}{\partial x}(-y) + (-y)\frac{\partial}{\partial y}(-y) + 0 = 0 - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$y = \frac{-1}{\rho}\frac{\partial p}{\partial y} \Rightarrow \frac{\partial p}{\partial y} = -\rho y$$

Pressure gradient vector $= \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} = -\rho x\hat{i} - \rho y\hat{j} = -\rho\left(x\hat{i} + y\hat{j}\right)$

So answer is option 'D'.

9. One-dimensional steady state heat conduction takes place through a solid whose cross-sectional area varies linearly in the direction of heat transfer. Assume there is no heat generation in the solid and the thermal conductivity of the material is constant and independent of temperature. The temperature distribution in the solid is

(A) Logarithmic (B) Quadratic (C) Linear (D) Exponential

Key: (A)

Sol: 1-D steady state with no heat generation

$$\frac{d}{dx}\left(-k\,A\frac{dT}{dx}\right)=0$$

$$\Rightarrow -kA\frac{dT}{dx}=C_1$$

$$\because A=Cx+B\left(\text{linear variation}\right)$$

$$\Rightarrow \int_1^2 dT=\int_1^2\frac{-C_1}{k\left(Cx+B\right)}dx$$

$$T_2-T_1=\frac{-C_1}{K}\ell n\left(\frac{Cx_2+B}{Cx_1+B}\right)$$

$\therefore$ Temperature distribution in the solid is logarithmic

10. Endurance limit of a beam subjected to pure bending decreases with

(A) decrease in the surface roughness and increase in the size of the beam

(B) increase in the surface roughness and decrease in the size of the beam

(C) increase in the surface roughness and increase in the size of the beam

(D) decrease in the surface roughness and decrease in the size of the beam

Key: (C)

Sol: $\sigma_e=k_a.k_b.k_c.k_d\,.\,\sigma'_e$

σ'_e – endurance strength, σ_e = corrected endurance strength

K_a = size factor, K_b = surface factor, K_c = load factor, K_d – temperature factor

With the increase in surface roughness, size factor etc the endurance strength drops.

11. Which one of the following modifications of the simple ideal Rankine cycle increases the thermal efficiency and reduces the moisture content of the steam at the turbine outlet?

(A) Decreasing the condenser pressure

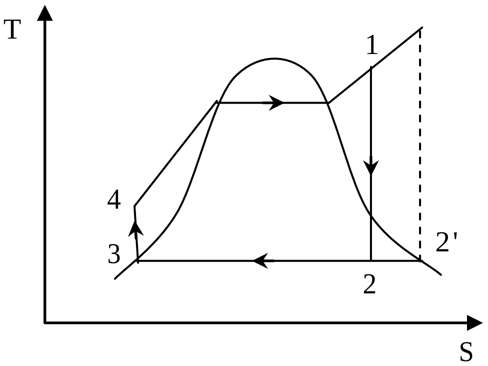
(B) Increasing the boiler pressure

(C) Decreasing the boiler pressure

(D) Increasing the turbine inlet temperature

Key: (D)

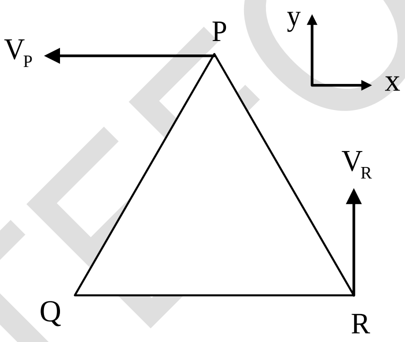
Sol:



Increasing the inlet temperature to the turbine improves of quality of steam at the outlet of the turbine.

This is the reason we use reheat cycle.

12. A rigid triangular body, PQR, with sides of equal length of 1 unit moves on a flat plane. At the instant shown, edge QR is parallel to the x-axis, and the body moves such that velocities of points P and R are V_P and V_R, in the x and y directions, respectively. The magnitude of the angular velocity of the body is



(A) $V_R/\sqrt{3}$ (B) $V_P/\sqrt{3}$ (C) $2V_P$ (D) $2V_R$

Key: (D)

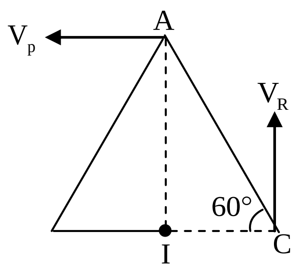
Sol: Locating the I-centre

$AI = AC \sin 60°$

$CI = AC \cos 60°$

$V_p = (AI)\,\omega \Rightarrow \omega = \frac{2}{\sqrt{3}} V_p$

$V_R = (CI)\omega \Rightarrow \omega = 2V_R \quad \therefore \text{Option}(D)$



13. For a simple compressible system, v, s, p and T are specific volume, specific entropy, pressure and temperature, respectively. As per Maxwell's relation, $\left(\frac{\partial v}{\partial s}\right)_P$ is equal to

(A) $\left(\frac{\partial T}{\partial p}\right)_s$ (B) $-\left(\frac{\partial T}{\partial p}\right)_p$ (C) $\left(\frac{\partial s}{\partial T}\right)_P$ (D) $\left(\frac{\partial p}{\partial v}\right)_T$

Key: (A)

Sol: $\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$

$dh = Tds + Vdp$

$dx = Mdy + Ndz$

Maxwell's relation can be obtained by using

$$\left(\frac{\partial M}{\partial Z}\right)_y = \left(\frac{\partial N}{\partial y}\right)_z \Rightarrow \left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial S}\right)_p$$

14. The most common limit gage used for inspecting the hole diameter is

(A) Snap gage (B) Plug gage (C) Ring gage (D) Master gage

Key: (B)

15. The directional derivative of the function $f(x,y) = x^2 + y^2$ along a line directed from (0, 0) to (1, 1), evaluated at the point x = 1, y = 1 is

(A) $4\sqrt{2}$ (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\sqrt{2}$

Key: (C)

Sol: Given, Scalar valued function $f(x,y) = x^2 + y^2$

Line directed from (0, 0) to (1, 1) is $(1-0)\hat{i} + (1-0)\hat{j}$

$\therefore$ Directional derivative of $f(x,y)$ at (1, 1) in the direction of $\hat{i}+\hat{j}$ is given by

i.e., $(\text{DD of } f)_{(1,1)}$ in $(\hat{i}+\hat{j})$ direction $= (\nabla f)_{(1,1)} \cdot \frac{(\hat{i}+\hat{j})}{|\hat{i}+\hat{j}|}$ ...(1)

$$\therefore \nabla f = \hat{i}\frac{\partial}{\partial x}(x^2+y^2) + \hat{j}\frac{\partial}{\partial y}(x^2+y^2)$$

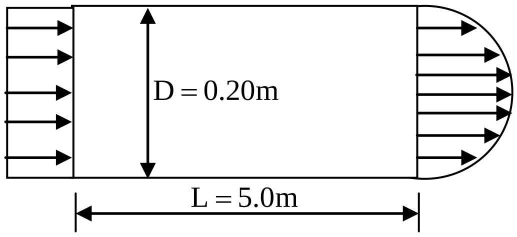
$$\Rightarrow \nabla f = \hat{i}(2x) + \hat{j}(2y) \Rightarrow (\nabla f)_{(1,1)} = 2\hat{i} + 2\hat{j}$$

$\therefore$ From (1), D.D of f = $(2\hat{i}+2\hat{j}) \cdot \frac{(\hat{i}+\hat{j})}{|\hat{i}+\hat{j}|} = \frac{2+2}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

16. Water enters a circular pipe of length L = 5.0 m and diameter D = 0.20 m with Reynolds number $Re_D = 500$. The velocity profile at the inlet of the pipe is uniform while it is parabolic at the exit. The Reynolds number at the exit of the pipe is _______.

Key: (500)

Sol:



Given that Reynold's number(Re_D) = 500

We know that Reynold's number for flow through pipes is,

$$Re_D = \frac{\rho V_{avg} D}{\mu}$$

ρ = density of fluid, V_{avg} = average velocity of fluid

D = diameter of pipe, μ = dynamic viscosity of fluid

$$V_{avg} = \frac{4Q}{\pi D^2}, \text{ where } Q = \text{discharge}$$

$$\therefore Re_D = \frac{\rho\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{\rho\left(\frac{4Q}{\pi D}\right)}{\mu}$$

So Reynold's number is a function of density, discharge, diameter and dynamic viscosity.

Since density, diameter and dynamic viscosities are same at entrance and exit and discharge is same hence the Reynold's number is also same at entrance and exit.

17. In matrix equation [A]{X} = {R},

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}.$$

One of the eigenvalues of matrix [A] is

(A) 8 (B) 16 (C) 15 (D) 4

Key: (B)

Sol: Given $[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{x\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix}$ and $\{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$

Clearly, $[A]\{x\}=\{R\} \Rightarrow \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix} \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix} = 16 \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix}$

$\therefore$ The above equation is of the form $AX=\lambda X$; where 'λ' is an eigen value

$\therefore$ One of the Eigen values of matrix must be '16'.

18. Sphere 1 with a diameter of 0.1 m is completely enclosed by another sphere 2 of diameter 0.4 m. The view factor F_{12} is

(A) 0.0625 (B) 0.5 (C) 1.0 (D) 0.25

Key: (C)

Sol: $d_1 = 0.1\,m$

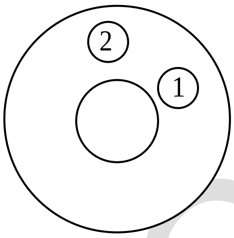
$d_2 = 0.4\,m$

$F_{1-1} = 0$

Summation rule

$F_{1-1} + F_{1-2} = 1$

$\Rightarrow F_{1-2} = 1 \quad (\because F_{1-1} = 0)$



19. In an electrical discharge machining process, the breakdown voltage across inter electrode gap (IEG) is 200V and the capacitance of the RC circuit is $50\mu F$. The energy (in J) released per spark across the IEG is ____

Key: (1)

Sol: IEG = 200

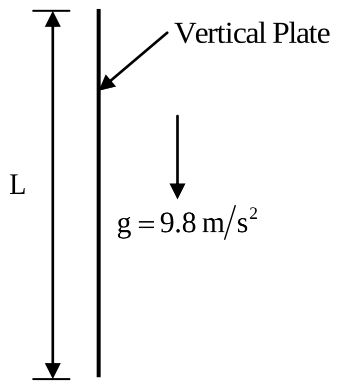
$C = 50\mu F$

Energy $(e) = ?$

$E = \frac{1}{2} CV^2$

$E = 1\,J$

20. A thin vertical flat plate of height L, and infinite width perpendicular to the plane of the figure, is losing heat to the surroundings by natural convection. The temperature of the plate and the surroundings, and the properties of the surrounding fluid, are constant. The relationship between the average Nusselt and Rayleigh numbers is given as $Nu = KRa^{1/4}$, where K is a constant. The length scales for Nusselt and Rayleigh numbers are the height of the plate. The height of the plate is increased to 16L keeping all other factors constant



If the average heat transfer coefficient for the first plate is h_1 and that for the second plate is h_2, the value of the ratio h_1/h_2 is___________.

Key: (2)

Sol: $N_u = K(R_a)^{1/4}$

$\Rightarrow N_u \propto (Ra)^{1/4}$ Rayleigh number $R_a = G_r P_r$

$\Rightarrow N_u \propto (Gr.Pr)^{1/4}$ As Prandtl number can be considered as a property. It remains constant

$$\Rightarrow N_u \propto (Gr)^{1/4}$$

$$\Rightarrow \frac{hL}{K} \propto \left(\frac{g\beta\Delta TL^3}{v^2}\right)^{1/4} \Rightarrow hL \propto L^{3/4}$$

$$\Rightarrow h \propto \frac{L^{3/4}}{L} \Rightarrow h \propto L^{\frac{3}{4}-1} \Rightarrow h \propto L^{-1/4}$$

$$\Rightarrow \frac{h_2}{h_1} = \left(\frac{L_1}{L_2}\right)^{1/4}$$

$$\text{and } \frac{h_1}{h_2} = \left(\frac{L_2}{L_1}\right)^{1/4} = \left(\frac{16L}{L}\right)^{1/4} = \left(2^4\right)^{1/4} \Rightarrow \frac{h_1}{h_2} = 2$$

21. A spur gear has pitch circle diameter D and number of teeth T. The circular pitch of the gear is

(A) $\frac{D}{T}$ (B) $\frac{2\pi D}{T}$ (C) $\frac{\pi D}{T}$ (D) $\frac{T}{D}$

Key: (C)

Sol: Circular pitch = $\pi m = \frac{\pi D}{T}$

22. An analytic function f(z) of complex variable z = x + iy may be written as f(z) = u(x, y) + iv(x, y). Then u(x, y) and v(x, y) must satisfy

(A) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

(B) $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

(C) $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

(D) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

Key: (D)

Sol: Given an analytic function $f(z)=u(x,y)+iv(x,y)$

If $f(z)=u+iv$ is analytic then u, v must satisfy Cauchy –Riemann equations i.e,

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \ \& \ \frac{\partial u}{\partial y}=\frac{-\partial v}{\partial x}$$

23. Hardenability of steel is a measure of

(A) the ability to retain its hardness when it is heated to elevated temperatures

(B) the ability to harden when it is cold worked

(C) the depth to which required hardening is obtained when it is austenitized and then quenched

(D) the maximum hardness that can be obtained when it is austenitized and then quenched

Key: (C)

24. The transformation matrix for mirroring a point in x-y plane about the line y = x is given by

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Key: (A)

25. The differential equation $\frac{dy}{dx}+4y=5$ is valid in the domain $0\le x\le 1$ with $y(0)=2.25$. The solution of the differential equation is

(A) $y=e^{-4x}+1.25$

(B) $y=e^{4x}+1.25$

(C) $y=e^{-4x}+5$

(D) $y=e^{4x}+5$

Key: (A)

Sol: Given differential equation $\frac{dy}{dx}+4y=5,\ 0\le x\le 1$ with $y(0)=2.25$

Clearly the above differential equation is linear, where P=4, Q=5

$\therefore$ I.F$=e^{\int 4dx}=e^{4x}$

Solution of equation (1) is

$$y\left(e^{4x}\right)=\int 5.e^{4x}.dx+C \Rightarrow ye^{4x}=5\frac{e^{4x}}{4}+C \qquad ...(2)$$

Given y=2.25 at x = 0

$$(2) \Rightarrow 2.25(1)=\frac{5}{4}+C \Rightarrow C=2.25-1.25=1 \Rightarrow C=1$$

$\therefore$ From (2), the required solution is

$$ye^{4x}=\frac{5}{4}e^{4x}+1$$

$$y=\frac{5}{4}+e^{-4x}$$

$$\Rightarrow y=e^{-4x}+1.25$$

Q. No. 26 to 55 Carry Two Marks Each

26. In an orthogonal machining with a single point cutting tool of rake angle 10°, the uncut chip thickness and the chip thickness are 0.125 mm and 0.22 mm, respectively. Using Merchant's first solution for the condition of minimum cutting force, the coefficient of friction at the chip-tool interface is _____ (round off to two decimal places).

Key: (0.74)

Sol: $\alpha=10°, t_1=0.125\text{mm } t_2=0.22\text{mm}$

$$\tan\phi=\frac{\cos 10°}{\frac{0.22}{0.125}-\sin 10°}=\frac{0.98}{1.76-0.1736}$$

$$\tan\phi=0.6177 \Rightarrow \phi=31.706°$$

For minimum cutting force

$$2\phi+\beta-\alpha=\frac{\pi}{2}=90°$$

$$2\times 31.706+\beta-10=90°$$

$$\beta=36.59°$$

$$\tan\beta=\mu=0.7423 \Rightarrow \mu=0.74$$

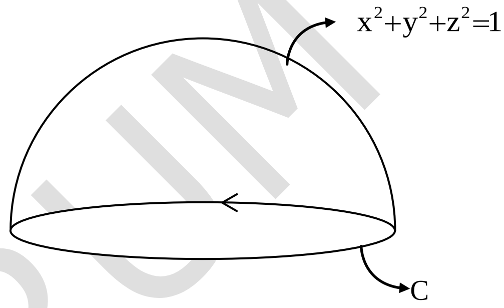
27. Given a vector $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3 k\right)$ and n as the unit normal vector to the surface of the hemisphere $\left(x^2 + y^2 + z^2 = 1; z \geq 0\right)$, the value of integral $\int (\nabla \times \vec{u}).n\, ds$ evaluated on the curved surface of the hemisphere S is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) π (D) $-\frac{\pi}{2}$

Key: (A)

Sol: **Method –I:**

Given $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{i} + z^3\hat{k}\right)$ and



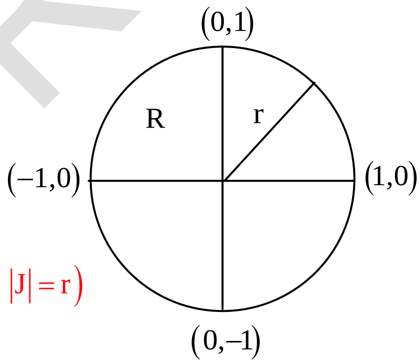
$\hat{n} \to$ Unit normal vector to the surface of the hemi-sphere

$x^2 + y^2 + z^2 = 1;\ z \geq 0.$

Using Stoke's theorem, we have

$$\int_s (\nabla \times \vec{u})\ .\ \hat{n}ds = \int_c \vec{u}\ .\ d\vec{r} = \int_c \left[-\frac{y^3}{3}dx + \frac{x^3}{3}dy + \frac{z^3}{3}dz\right]$$

$$= \oint_c \underbrace{-\frac{y^3}{3}}_{M}dx + \underbrace{\frac{x^3}{3}}_{N}dy \quad [\because z = 0 \text{ on 'c'}]$$



$$= \iint_R \left(x^2 + y^2\right)dx\, dy \left(\text{Using Green's theorem}\right) \to$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{1} r^2\ r\, dr\, d\theta \left(\text{Using polar co-ordination } x^2 + y^2 = r^2, |J| = r\right)$$

$$= \int_{\theta=0}^{2\pi}\left[\int_{r=0}^{1} r^3 dr\right] d\theta = \int_{\theta=0}^{2\pi}\left[\frac{r^4}{4}\right]_0^1 d\theta = \frac{1}{4}[\theta]_0^{2\pi} = \frac{\pi}{2}$$

Method –II:

Given, $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$

$$\nabla \times \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{y^3}{3} & \frac{x^3}{3} & \frac{z^3}{3} \end{vmatrix} = \hat{k}\left[x^2 + y^2\right]$$

$$\therefore \hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = x\hat{i} + y\hat{j} + z\hat{k} \quad \left(\because \nabla\phi = \hat{i}(2x) + \hat{j}(2y) + \hat{k}(2z)\right)$$

$$|\nabla\phi| = \sqrt{4x^2 + 4y^2 + 4z^2} = \sqrt{4(1)} = 2$$

$$\iint (\nabla \times \vec{u})\ .\hat{n}ds = \iint z\left(x^2 + y^2\right) ds \qquad ...(1)$$

Let us parameterize 'S' in spherical coordinates, with colatitude $0 \le \phi \le \frac{\pi}{2}$ and longitude $0 \le \theta \le 2\pi$;

$x = \sin\phi\cos\theta;\ y = \sin\phi\sin\theta;\ z = \cos\phi;\ ds = \sin\phi\, d\theta d\phi$

$$\therefore \iint (\nabla \times \vec{u}).\hat{n}ds = \int_{\phi=0}^{\pi/2}\int_{\theta=0}^{2\pi} \cos\phi\left[\sin^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta\right]\sin\phi\, d\theta d\phi$$

$$= \int_{\phi=0}^{\pi/2}\int_{\theta=0}^{2\pi}\left[\cos\phi.\sin^3\phi\, d\theta d\phi\right]\ \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \int_{\phi=0}^{\pi/2}\cos\phi\sin^3\phi\left[\theta\right]_0^{2\pi}.d\phi = (2\pi)\int_{\phi=0}^{\pi/2}\cos\phi\sin^3\phi.\, d\phi = 2\pi\left(\frac{(3-1)}{4\times 2}\right);$$

$$\left[\because \int_0^{\pi/2}\sin^n x.\cos^n x dx = \frac{\left[(m-1)(m-3)....\right]\times\left[(n-1)(n-3).....\right]}{(m+n)(m+n-2)......}k \right.$$
$$\left. \text{Where } k = \pi/2;\ \text{if } n\,\&\,m \text{ both are even; } k = 1;\ \text{otherwise.}\right]$$

$$= 2\pi \times \frac{2}{4\times 2}$$

$$\Rightarrow \iint(\nabla\times\vec{u}).\hat{n}\, ds = \frac{\pi}{2}$$

28. The derivative of $f(x) = \cos(x)$ can be estimated using the approximation

$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$. The percentage error is calculated as

$\left(\frac{\text{Exact value} - \text{Aprroximate value}}{\text{Exact value}}\right)\times 100$. The percentage error in the derivative of

$f(x)$ at $x = \pi/6$ radian, choosing $h = 0.1$ radian, is

(A) >5% (B) > 0.1% and <1% (C) <0.1% (D) >1 % and <5%

Key: (B)

Sol: Given $f(x) = \cos x$

$\Rightarrow f'(x) = -\sin x$

$\Rightarrow f'(x)\big|_{x=\pi/6} = -\sin\left(\frac{\pi}{6}\right) = \frac{-1}{2} = -(0.5)$

$\therefore$ Exact value of the derivative = -0.5 ...(1)

Approximate value of derivative

$$f'(x)=\frac{f(x+h)-f(x-h)}{2h}\Rightarrow f'(x)=\frac{\cos(x+h)-\cos(x-h)}{2h}$$

$$\Rightarrow f'\left(\frac{\pi}{6}\right)=\frac{\cos\left(\frac{\pi}{6}+0.1\right)-\cos\left(\frac{\pi}{6}-0.1\right)}{2(0.1)}\ ;\ \text{since } h=0.1$$

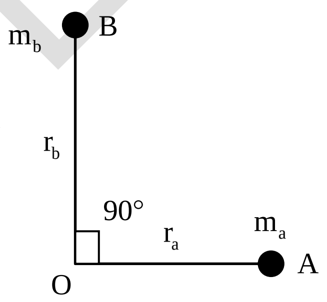
$$=\frac{\left[\cos\left(\frac{\pi}{6}\right).\cos(0.1)\ -\sin\left(\frac{\pi}{6}\right).\sin(0.1)\right]-\left[\cos\left(\frac{\pi}{6}\right)\cos(0.1)+\sin\left(\frac{\pi}{6}\right)\sin(0.1)\right]}{2(0.1)}$$

$$=\frac{-2\sin\left(\frac{\pi}{6}\right).\sin(0.1)}{2(0.1)}=\frac{-[0.5]\sin(0.1)}{(0.1)}\approx-0.4992$$

$$\therefore f'\left(\frac{\pi}{6}\right)\approx-0.4992\ (\text{Approximate value})\qquad ...(2)$$

$$\therefore \text{Percentage error}=\left[\frac{-0.5-(-0.4992)}{-0.5}\right]\times100=0.16\%\ \text{ie.}>0.1\%\ \text{and}<1\%$$

29. Two masses A and B having mass m_a and m_b, respectively, lying in the plane of the figure shown, are rigidly attached to a shaft which revolves about an axis through O perpendicular to the plane of the figure. The radii of rotation of the masses m_a and m_b are r_a and r_b, respectively. The angle between lines OA and OB is 90°. If m_a = 10 kg, $m_b=20$ kg $r_a=200$ mm and $r_b=400$mm, then the balance mass to be placed at a radius of 200 mm is ____ kg (round off to two decimal places)



Key: (41.231)

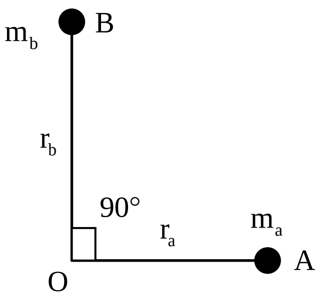
Sol: Method-1:

$m_a=10\text{kg},\ m_b=20\text{kg},\ r_a=200\text{mm},=0.2\text{mts},$
$r_b=400\text{mm}=0.4\text{mts},\quad r_r=200\text{mm}=0.2\text{mts}.$

$$(mr)_{\text{resultant}}=\sqrt{(mr)_a^2+(mr)_b^2}=\sqrt{(10\times0.2)^2+(20\times0.4)^2}=8.24$$

$$m_{\text{resultant}}=\frac{8.24}{0.2}=41.231\text{ kg}$$

Method-2:



$m_a = 10kg \quad r_a = 200mm$

$m_b = 20kg \quad r_b = 400mm$

at r = 200 mm m = ? for balancing

$\Sigma F_x = 0$

$m_a r_a + m_b r_b \cos(90°) + mr\cos\theta = 0$

$m_a r_a + 0 + mr\cos\theta = 0$

$\Sigma F_y = 0$

$m_b r_b + mr\sin\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = \frac{m_b r_b}{m_a r_a}$$

$$\tan\theta = \frac{20\times 0.4}{10\times 0.2}$$

$\theta = \tan^{-1} 4 \Rightarrow \theta = 75.964^0$

$$m = \frac{m_b r_b}{r\sin\theta} = 41.23kg$$

30. A through hole is drilled in an aluminum alloy plate of 15 mm thickness with a drill bit of diameter 10 mm, at a feed of 0.25 mm/rev and a spindle speed of 1200 rpm. If the specific energy required for cutting this material is $0.7\ N-m/mm^3$, the power required for drilling is ___ W (round off to two decimal places).

Key: (274.889)

Sol: $t = 15mm$

$d = 10mm$

$f = 0.25 mm/rev$

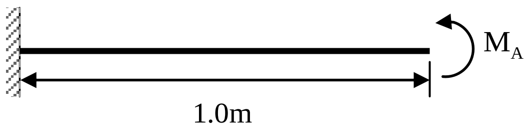
$N = 1200rpm$

Specific energy required = $0.7\ Nm/mm^2$

Power = ?

$$\text{Power} = \frac{\pi}{4}D^2 f\frac{N}{60}\times(SER) = 274.889\ W$$

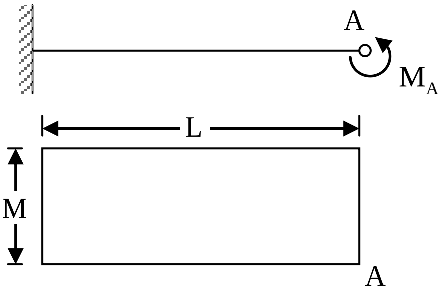
31. A horizontal cantilever beam of circular cross-section, length 1.0 m and flexural rigidity $EI = 200\ N-m^2$ is subjected to an applied moment $M_A = 1.0$ N-m at the free end as shown in the figure.



The magnitude of the vertical deflection of the free end is ____ mm (round off to one decimal place)

Key: (2.5)

Sol:

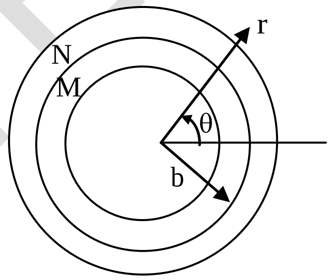


$EI = 200N - m^2, \quad L = 1.0mts, \quad M_A = 1.0N - mts$

$$\delta = \frac{\text{moment of Area under B.M.D}}{EI} = \frac{ML}{EI} \times \frac{L}{2} = \frac{ML^2}{2EI}$$

$$= \frac{1 \times 1^2}{2 \times 200} = 2.5mm$$

32. Consider two concentric circular cylinders of different materials M and N in contact with each other at r = b, as shown below. The interface at r = b is frictionless. The composite cylinder system is subjected to internal pressure P. Let $\left(u_r^M, u_\theta^M\right)$ and $\left(\sigma_{rr,}^M\ \sigma_{\theta\theta}^M\right)$ denote the radial and tangential displacement and stress components, respectively, in material M. Similarly $\left(u_r^N, u_\theta^N\right)$ and $\left(\sigma_{rr}^N, \sigma_{\theta\theta}^N\right)$ denote the radial and tangential displacement and stress components, respectively, in material N. The boundary condition that need to be satisfied at the frictionless interface between the two cylinders are:



(A) $u_r^M = u_r^N$ and $\sigma_{rr}^M = \sigma_{rr}^N$ and $u_\theta^M = u_\theta^N$ and $\sigma_{\theta\theta}^M = \sigma_{\theta\theta}^N$

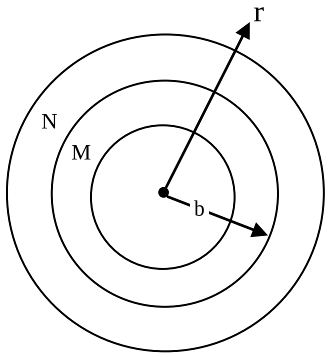
(B) $u_\theta^M = u_\theta^N$ and $\sigma_{\theta\theta}^m = \sigma_{\theta\theta}^N$ only

(C) $\sigma_{rr}^M = \sigma_{rr}^N$ and $\sigma_{\theta\theta}^M = \sigma_{\theta\theta}^N$ only

(D) $u_r^M = u_r^N$ and $\sigma_{rr}^M = \sigma_{rr}^N$ only

Key: (D)

Sol:

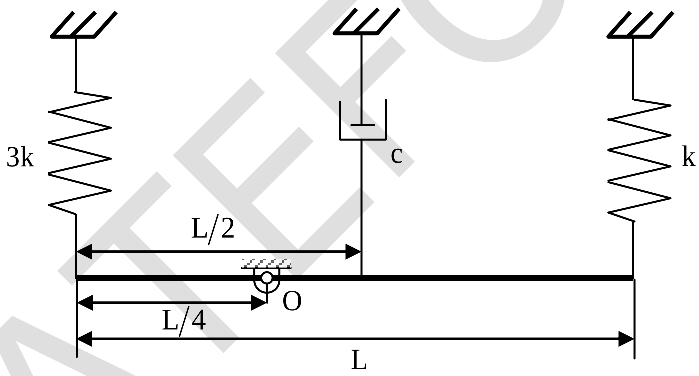


At the radial location r=b, there will be a contact pressure and element at interface of cylinder M and N the radial displacement will be same

$u_r^M = u_r^N$, $\sigma_{RR}^M = \sigma_{RR}^N$

$u_\theta^M \neq u_\theta^N$, $\sigma_{\theta\theta}^M \neq \sigma_{\theta\theta}^N$

33. A slender uniform rigid bar of mass m is hinged at O and supported by two springs, with stiffnesses 3k and k, and a damper with damping coefficient c, as shown in the figure. For the system to be critically damped, the ratio $c/\sqrt{km}$ should be



(A) $4\sqrt{7}$ (B) 4 (C) $2\sqrt{7}$ (D) 2

Key: (A)

Sol: $I_C = \frac{mL^2}{12}$, $I_0 = I_C + mr^2 \Rightarrow \frac{mL^2}{12} + m\left(\frac{L}{4}\right)^2 = \frac{7mL^2}{48}$

Taking moment about 0, $\Sigma M_o = 0$

$$I_0\ddot{\theta} + k\left(\frac{3L}{4}\theta\right)\left(\frac{3L}{4}\right) + 3k\left(\frac{L}{4}\theta\right)\left(\frac{L}{4}\right) + C\left(\frac{L}{4}\dot{\theta}\right)\left(\frac{L}{4}\right) = 0$$

$$\frac{7mL^2}{48}\ddot{\theta} + \frac{CL^2}{16}\dot{\theta} + \frac{12kL^2}{16}\theta = 0$$

$$\frac{7m}{48}\ddot{\theta} + \frac{C}{16}\dot{\theta} + \frac{12k}{16}\theta = 0$$

Critical damping, the roots of above differential equation are real and equal so we can use

$\because b^2 - 4ac = 0$

$a = \frac{7m}{48},\ b = \frac{C}{16},\ c = \frac{12k}{16}$

$(\text{Critical damping})C^2 = 16 \times 7mk$

$C = 4\left(\sqrt{7mk}\right) \Rightarrow \frac{C}{\sqrt{mk}} = 4\sqrt{7}$

34. An air standard Otto cycle has thermal efficiency of 0.5 and the mean effective pressure of the cycle is 1000 kPa. For air, assume specific heat ratio $\gamma = 1.4$ and specific gas constant R = 0.287 kJ/kg.K. If the pressure and temperature at the beginning of the compression stroke are 100 kPa and 300 K, respectively, then the specific net work output of the cycle is ___ kJ/kg (round off to two decimal places).

Key: (708.6)

Sol: $\eta_{OHO} = 0.5$

$P_m = 1000KPa \quad P_1 = 100KPa$

$\gamma = 1.4 \quad T_1 = 300K$

$R = 0.287\ KJ/kgk$

$$\eta = 1 - \frac{1}{(r_c)^{\gamma-1}}$$

$$0.5 = 1 - \frac{1}{(r_c)^{1.4-1}}$$

$$\frac{1}{(r_c)^{0.4}} = 0.5 \Rightarrow r_c = \left(\frac{1}{0.5}\right)^{\frac{1}{0.4}} = 5.65$$

$P_1V_1 = mRT$

$$\frac{V_1}{m} = \frac{RT_1}{P_1}$$

$$V_1 = \frac{0.287 \times 300}{100} = 0.861\,m^3/kg$$

$$\therefore r_c = \frac{V_1}{V_2}$$

$$\Rightarrow V_2 = \frac{V_1}{r_c} = \frac{0.861}{5.65}$$

$\therefore V_2 = 0.1524\,m^3/kg$ swept volume

$V_s = V_1 - V_2$

$$P_m = \frac{w.D}{V_1 - V_2} \Rightarrow W.D = P_m \times (V_1 - V_2)$$

$$= 1000 \times (0.681 - 0.1524) = 708.6\,KJ/kg$$

35. An idealized centrifugal pump (blade outer radius of 50mm) consumes 2kW power while running at 3000 rpm. The entry of the liquid into the pump is axial and exit from the pump is radial with respect to impeller. If the losses are neglected, then the mass flow rate of the liquid through the pump is ________kg/s (round off to two decimal places).

Key: (8.1057)

Sol: From the given data, since the inlet flow is axial, whirl component at inlet is zero $V_{W_1} = 0$

And also because of radial discharge whirl component exit is $V_{W_2} = u_2$

Power input to the pump is P=2kW and N=3000rpm and $d_2 = 2r_2 = 100mm = 0.1mm$

$$\therefore V_{W_2} = u_2 = \frac{\pi \times 0.1 \times 3000}{60} = 5\pi m/sec$$

Power input to the pump 'P' = $\dot{m}\left(V_{W_2}u_2 - V_{W_1}u_1\right)$

where $\dot{m} =$ mass flow rate of liquid in kg/sec

$$2000 = m((5\pi)(5\pi)) \quad (\because V_{w_1} = 0) \Rightarrow \dot{m} = 8.1057 kg/sec$$

36. A ball of mass 3 kg moving with a velocity of 4 m/s undergoes a perfectly-clastic direct-central impact with a stationary ball of mass m. After the impact is over, the kinetic energy of the 3 kg ball is 6J. The possible value (s) of m is/are

(A) 6 kg only (B) 1kg, 9 kg (C) 1 kg, 6 kg (D) 1 kg only

Key: (B)

Sol: $m = 3kg,\ V_1 = 4m/sec,\ m^2 = m\ \ u_2 = 0\ ,\ kE_1 = 65, e = 1$

Conservation of Momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(3\times 4) + (m_2 \times 0) = 3v_1 + mv_2$$
$$3v_1 + mv_2 = 12 \qquad ...(1)$$
$$e = \frac{v_2 - v_1}{u_1 - u_2} = 1 \Rightarrow v_2 - v_1 = 4 \qquad ...(2)$$

Conservation of Energy

$$(KE_1)_i + (KE_2)_i = (KE_1)_f + (KE_2)_f$$
$$\left(\frac{1}{2}\times 3\times 4^2\right) + 0 = 6 + \frac{1}{2}mv_2^2 \Rightarrow mv_2^2 = 36$$

From (1) and (2)

$$3(v_2 - 4) + mv_2 = 12$$
$$3v_2 + mv_2 = 24 \Rightarrow v_2 = \left(\frac{24}{3+m}\right)$$
$$m\left(\frac{24}{3+m}\right)^2 = 36$$

$$\Rightarrow 576m = \left(9 + m^2 + 6m\right)36$$
$$36m^2 - 360m + 324 = 0 \Rightarrow m^2 - 10m + 9 = 0$$
$$\left(m-9\right)\left(m-1\right) = 0 \Rightarrow m = 1,9kg$$

37. The annual demand of valves per year in a company is 10,000 units. The current order quantity is 400 valves per order. The holding cost is Rs. 24 per valve per year and the ordering cost is Rs. 400 per order. If the current order quantity is changed to Economic order quantity, then the saving in the total cost of inventory per year will be Rs____ (round of to two decimal places).

Key: (943.60)

Sol:
$$D = 1000 \text{ units}$$
$$Q = 400 \text{ Values / order}$$
$$C_n = 24\,\text{rs/value / year}$$
$$C_0 = 400 \text{ Rs/order}$$
$$EOQ = \sqrt{\frac{2DCo}{C_n}}$$
$$= \sqrt{\frac{2 \times 10000 \times 400}{24}}$$
$$Q^* = EOQ = 577.35$$
$$TIC(Q) = \sqrt{2DC_oC_n} = \sqrt{2 \times 10000 \times 400 \times 24} = 13856$$
$$TIC(Q) = \frac{D}{Q}C_0 + \frac{Q}{2}C_n$$
$$= \frac{10000}{400} \times 400 + \frac{400}{2}24$$
$$= 10000 + 4800 = 14800$$
$$\text{saving} = 14800 - 13856.41 = Rs.943.60$$

38. Water flowing at the rate of 1 kg/s through a system is heated using an electric heater such that the specific enthalpy of the water increases by 2.50 kJ/kg and the specific entropy increases by 0.007 kJ/kg K. The power input to the electric heater is 2.50 kW. There is no other work or heat interaction between the system and the surroundings, Assuming an ambient temperature of 300 K, the irreversibility rate of the system is _______kW (round off to two decimal places).

Key: (2.1)

Sol:
$$\dot{m} = 1kg/s$$
$$h_2 - h_1 = 2.5 \text{ kJ/kg},\ S_2 - S_1 = 0.007 \text{ kj/kg k}$$
$$w = 2.5kW,\ T_0 = 300K$$
$$\text{irreversibility } (I) = T_0(\Delta S) = 300 \times 0.007 = 2.1 \text{ kJ/kg}$$
$$\therefore I = m \times 2.1 = 1 \times 2.1 = 2.1 \text{ kW}$$

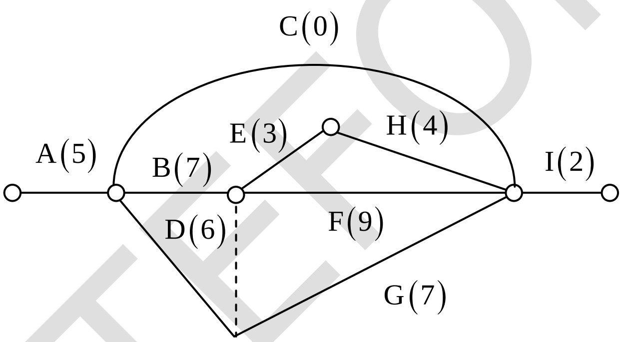
39. The activities of a project, their duration and the precedence relationship are given in the table. For example, in a precedence relationship "X <Y, Z" means that X is predecessor of activities Y and Z. The time to complete the activities along the critical path is _______weeks,

Activity	Duration (weeks)	Precedence Relationship
A	5	A<B, C,D
B	7	B<E,F,G
C	10	C<I
D	6	D <G
E	3	E<H
F	9	F<I
G	7	G<I
H	4	H<I
I	2	……

(A) 21 (B) 23 (C) 17 (D) 25

Key: (B)

Sol:



$ACI = 5 + 10 + 2 = 17 \text{days}$

$ABEHI = 5 + 7 + 3 + 4 + 2 = 21 \text{days}$

$ABFI = 5 + 7 + 9 + 2 = 23 \text{ days}$

$ABGI = 5 + 7 + 7 + 2 = 21 \text{days}$

Time required = 23 days

40. A differential equation is given as

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4.$$

The solution of the differential equation in terms of arbitrary constants C_1 and C_2 is

(A) $y = C_1x^2 + C_2x + 4$ (B) $y = \frac{C_1}{x^2} + C_2x + 4$

(C) $y = \frac{C_1}{x^2} + C_2x + 2$ (D) $y = C_1x^2 + C_2x + 2$

Key: (D)

Sol: Given differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=4 \quad \text{...Cauchy Euler Linear D.E}$$

$$\Rightarrow\left[x^2D^2-2xD+2\right]y=4 \qquad \text{...(1)}$$

$$\text{Consider } xD=\theta;\ x^2D^2=\theta(\theta-1) \text{ where } \theta=\frac{d}{dz}\ \&\ x=e^z$$

$$\therefore \text{From (1)}, \left[\theta(\theta-1)-2\theta+2\right]y=4$$

$$\Rightarrow\left[\theta^2-\theta-2\theta+2\right]y=4$$

$$\Rightarrow\left[\theta^2-3\theta+2\right]y=4 \qquad \text{...(2)}$$

$$\text{A.E } m^2-3m+2=0\Rightarrow(m-2)(m-1)=0$$

$$\Rightarrow m=2,1\rightarrow \text{real \& distinct}$$

$$\therefore y_C=C_1e^{2z}+C_2e^z \qquad \text{...(3)}$$

$$\therefore y_p=\frac{1}{\theta^2-3\theta+2}4.e^{0\times z}$$

substitute $\theta=0$; then

$$y_p=\frac{1}{2}4\Rightarrow y_p=2$$

$$\therefore \text{Complete solution is } y=y_C+y_p$$

$$\Rightarrow y=C_1e^{2z}+C_2e^z+2\Rightarrow y=C_1x^2+C_2x+2\left[\because x=e^z\right]$$

41. Water flows through two different pipes A and B of the same circular cross-section but at different flow rates. The length of pipe A is 1.0 m and that of pipe B is 2.0 m. The flow in both the pipes is laminar and fully developed. If the frictional head loss across the length of the pipes is same, the ratio of volume flow rate Q_B/Q_A is (round off to two decimal places).

Key: (0.5)

Sol: Given that, diameter of pipe 'A' (d_A) = diameter of pipe 'B' (d_B)

Length of pipe 'A' is $(\ell_A)=1\text{m}$

Length of pipe 'B' is $(\ell_B)=2\text{m}$

Frictional head loss in laminar flow is $h_f=\dfrac{32\mu v\ell}{\rho g d^2}$

It is mentioned in the question as $(h_f)_A=(h_f)_B$

$$\Rightarrow \frac{32\mu V_A \ell_A}{\rho g d_A^2} = \frac{32\mu V_B \ell_B}{\rho g d_B^2} \qquad (\because d_A = d_B)$$

$$V_A \ell_A = V_B \ell_B$$

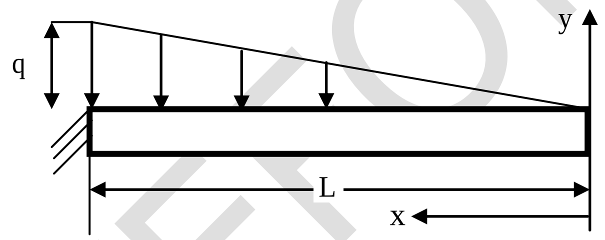
$$V_A(1) = V_B(2) \Rightarrow \frac{V_A}{V_B} = 2 \Rightarrow \frac{V_B}{V_A} = \frac{1}{2}$$

Flow rate pipe 'A' is $Q_A = V_A \frac{\pi}{4} d_A^2$

Flow rate in pipe 'B' is $Q_B = V_B \frac{\pi}{4} d_B^2$

$$\frac{Q_B}{Q_A} = \frac{(V_B)\left(\frac{\pi}{4}d_B^2\right)}{(V_A)\left(\frac{\pi}{4}d_A^2\right)} = \frac{V_B}{V_A} = \frac{1}{2} = 0.5$$

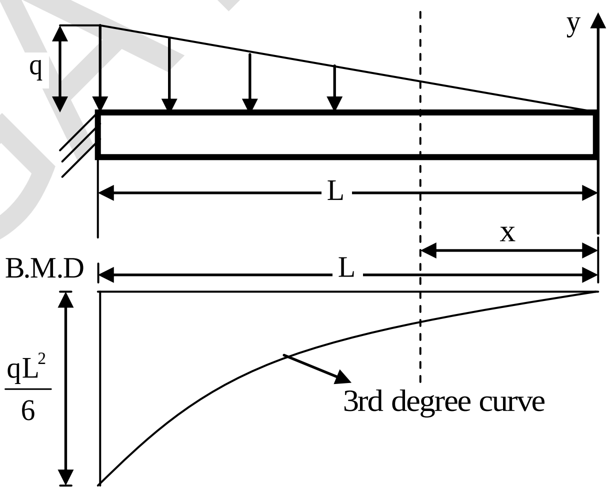
42. A prismatic, straight, elastic, cantilever beam is subjected to a linearly distributed transverse load as shown below. If the beam length is L, Young's modulus E, and are moment of inertia I, the magnitude of the maximum deflection is



(A) $\frac{qL^4}{10EI}$ (B) $\frac{qL^4}{15EI}$ (C) $\frac{qL^4}{60EI}$ (D) $\frac{qL^4}{30EI}$

Key: (D)

Sol:



$$M_{x-x} = \frac{1}{2} \times x \times \frac{qx}{L} \times \frac{x}{3} = \frac{qx^3}{6L}$$

$$M_A = \frac{1}{2} \times q \times L \times \frac{L}{3} \text{ (at A, } x = L)$$

$$\text{Deflection} = \frac{\text{Moment of area under BMD about free end}}{EI}$$

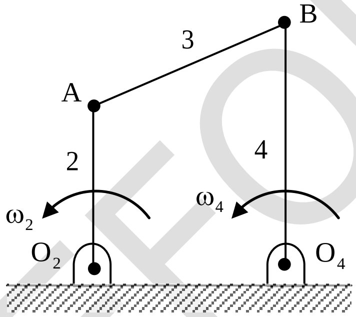
$$\left[\begin{array}{l} \text{Area of spandral} = \frac{1}{n+1} bh \\ \text{centriod of spandral about apex}(\bar{x}) = \frac{n+1}{n+2} b \end{array}\right] n = 3$$

$$\delta = \frac{A\bar{x}}{EI} = \left(\frac{1}{3+1}\right)\left(\frac{qL^2}{6}\right) . L . \left(\frac{3+1}{3+2}\right)\frac{L}{EI} = \frac{qL^4}{30EI}$$

43. A four bar mechanism is shown in the figure. The link numbers are mentioned near the links, input link 2 is rotating anticlockwise with a constant angular speed ω_2. Length of different links are:

$O_2O_4 = O_2A = L,$

$AB = Q_4B = \sqrt{2}L$

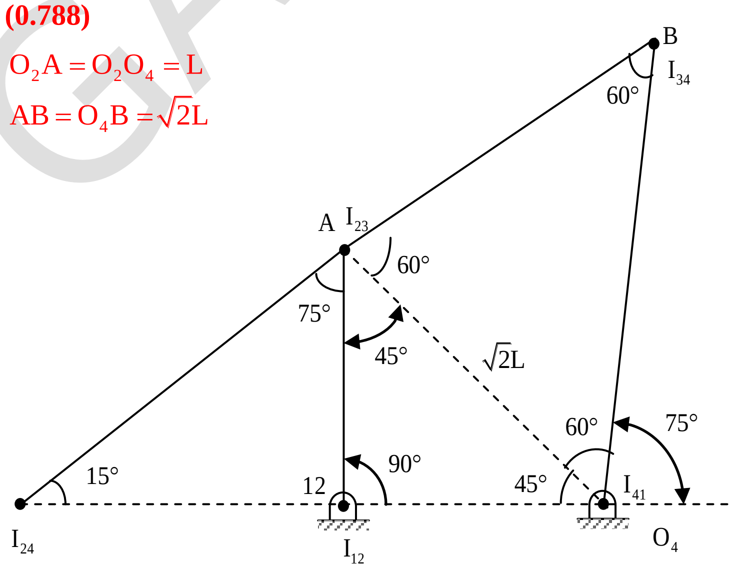


The magnitude of the angular speed of the output link 4 is ω_4 at the instant when link 2 makes an angle of 90° with O_2O_4 as shown. The ratio $\frac{\omega_4}{\omega_2}$ is ______(round off to two decimal places).

Key: (0.788)

Sol: $O_2A = O_2O_4 = L$

$AB = O_4B = \sqrt{2}L$



$$I_{24}\ I_{12}\omega_2 = I_{24}\ I_{41}\omega_4$$
$$I_{24}\ I_{12} = L\tan 75° \left(\text{from}\ \Delta^{le}\ I_{24}\ I_{12}\ I_{23}\right)$$
$$I_{24}\ I_{41} = L + L\tan 75°$$
$$\frac{\omega_4}{\omega_2} = \frac{I_{24}\ I_{12}}{I_{24}\ I_{41}} = \frac{L\tan 75°}{L + L\tan 75^0} = 0.788$$

44. A gas tungsten are welding operation is performed using a current of 250 A and an arc voltage of 20 V at a welding speed of 5 mm/s. Assuming that the arc efficiency is 70%the net heat input per unit length of the weld will be______ kJ/mm (round off to one decimal place).

Key: (0.7)

Sol: Given that, V = 20V, I = 250A

Welding speed = 5 mm/sec

Arc efficiency = 70% = 0.7

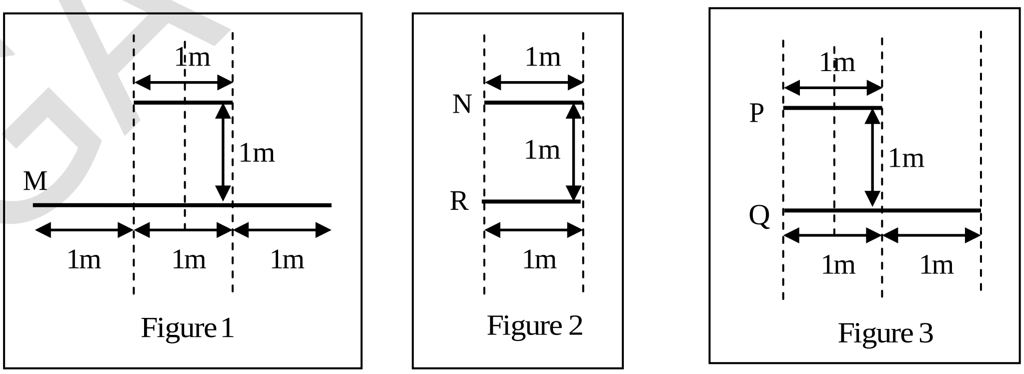
Power given to the welding operation $= VI = (20)(250) = 5000\ \text{watts} = 5000\ \text{Joules/sec}$

Since arc efficiency is 70%, net heat input will be $= 0.7 \times 5000 = 3500\ \text{Joules/sec}$

Net heat input per unit length of the weld $= \dfrac{\text{net heat input}}{\text{welding speed}}$

$$= \frac{3500(J/sec)}{50(mm/sec)} = 700\ J/mm = 0.7\ kJ/mm$$

45. Three sets of parallel plate LM, NR and PQ are given in Figures 1, 2 and 3. The view factor F_{IJ} is defined as the fraction of radiation leaving plate I that is intercepted by plate J. Assume that the values of F_{LM} and F_{NR} are 0.8 and 0.4 respectively. The value of F_{PQ} (round off to one decimal place) is________.



Key: (0.6)

Sol: From the figures we can say that view factor between o parallel $1m$ plates $= 0.4 = F_{NR}$

View factor between one $1m$ plate to another two equally inclined plates $= F_{LM} - F_{NR} = 0.8 - 0.4 = 0.4$

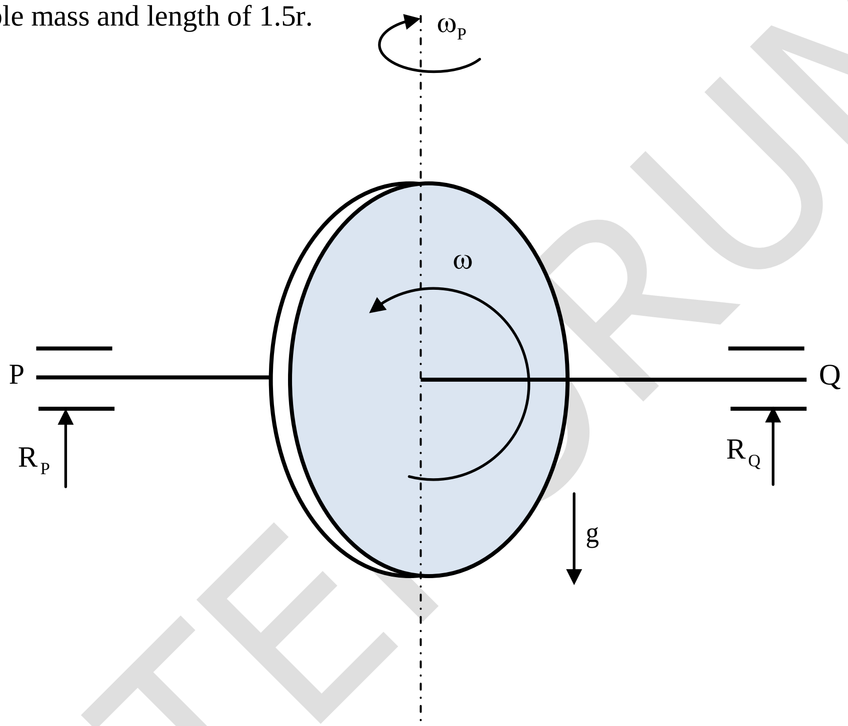
View factor between one $1m_4$ plate to and this equally inclined.

$$\text{Plate} = \frac{0.4}{2} = 0.2$$

$\therefore$ For figure '3' view factor form one 1m plate to another parallel plate and another equally inclined plate is

$F_{PQ} = 0.4 + 0.2 = 0.6$

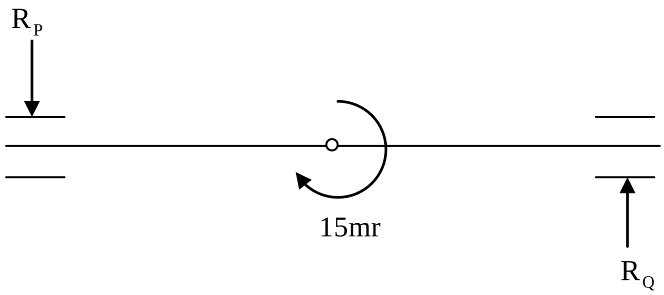
46. A uniform disc with radius r and a mass of m kg is mounted centrally on a horizontal axle of negligible mass and length of 1.5r.



The disc spins counter-clockwise about the axle with angular speed ω, when viewed from the right-hand side bearing Q, a The axle processes about a vertical axis at $\omega_p = \omega/10$ in the clockwise direction when viewed from above. Let R_p and R_Q (positive upwards) be the resultant reaction forces due to the mass and the gyroscopic effect, at bearings P and Q, respectively. Assuming $\omega^2 r = 300\,m/s^2$ and $g = 10\,m/s^2$, the ratio of the larger to the smaller bearing reaction force (considering appropriate signs) is________

Key: (-3)

Sol:



$\omega_p = \frac{\omega}{10}, \ \omega^2 r = 300\text{m/sec}^2, \ g = 10\text{m/sec}^2, \ \ell = 1.5r$

Gyroscope couple, $C = I\omega\omega_p$

$C = \left(\frac{mr^2}{2}\right)\omega.\omega_p$

$= \left(\frac{mr}{2}\right).\frac{r\omega^2}{10} = \frac{mr}{2} \times \frac{300}{10} = 15mr$

Reaction due to weight, $R_P = R_Q = \frac{mg}{2} = \frac{10m}{2} = 5m$ (due to symmetry)

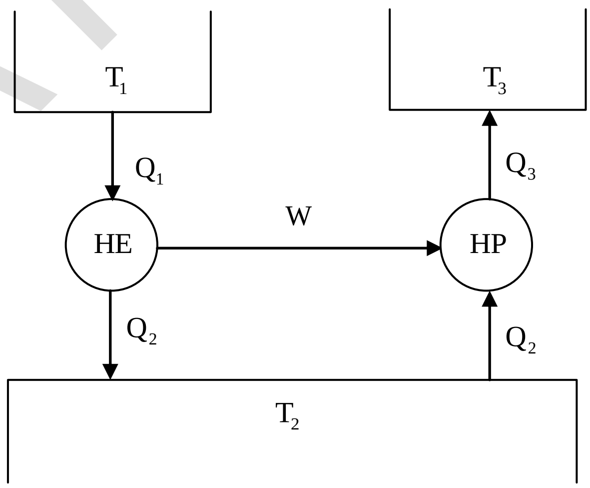
Reaction due to gyroscopic couple, $R_Q = \frac{15MR}{1.5R} = 10m, \ R_P = -10m$

Now net reaction, $R_Q = 10m + 5m = 15m$

$R_P = 5m - 10m = -5m$

$\frac{R_Q}{R_P} = \frac{15m}{-5m} = -3$

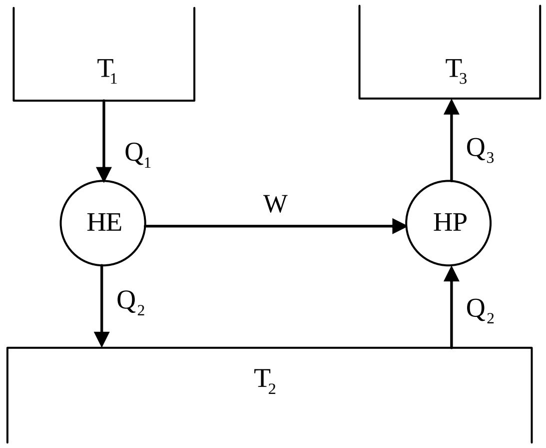
47. The figure shows a heat engine (HE) working between two reservoirs. The amount of heat (Q_2) rejected by the heat engine is drawn by a heat pump (HP). The heat pump receives the entire work output (W) of the heat engine. If temperatures, $T_1 > T_3 > T_2$, then the relation between the efficiency (η) of the heat engine and the coefficient and the coefficient of performance (COP) of the heat pump is



(A) $COP = \eta$ (B) $COP = \eta^{-1} - 1$ (C) $COP = \eta^{-1}$ (D) $COP = 1 + \eta$

Key: (C)

Sol:



$T_1 > T_3 > T_2 \text{ (Given)}$

$COP_{H.P} = \frac{Q_3}{W}$

$W = Q_1 - Q_2 \Rightarrow Q_2 = Q_1 - W$

$\eta = \frac{W}{Q_1} \text{ or } 1 - \frac{Q_2}{Q_1} \Rightarrow W = \eta \times Q_1$

$\text{Also } Q_3 = Q_2 + W$

$COP_{H.P} = \frac{Q_2 + W}{W}$

$= \frac{Q_2}{W} + 1 = \frac{Q_1 - W}{W} + 1 = \frac{Q_1}{W} - 1 + 1$

$\therefore COP = \eta^{-1}$

48. The aerodynamic drag on a sports car depends on its shape. The car has a drag coefficient of 0.1 with the windows and the roof closed. With the windows and the roof open, the drag coefficient becomes 0.8. The car travels at 44 km/h with the windows and roof closed. For the same amount of power needed to overcome the aerodynamic drag, the speed of the car with the windows and roof open (round off to two decimal places), is ____ km/h. (The density of air and the frontal area may be assumed to be constant.)

Key: (22)

Sol: Co-efficient of drag with doors and roof closed $(C_d)_C = 0.1$

Co-efficient of drag with doors and roof open $(C_d)_O = 0.8$

Velocity of car with doors and roof closed $V_C = 44 \text{km/sec}$

Velocity of car with doors and roof open $V_O = ?$

Power needed in overcoming aerodynamic drag with doors and roof closed =

Power needed in overcoming aerodynamic with doors and roof open

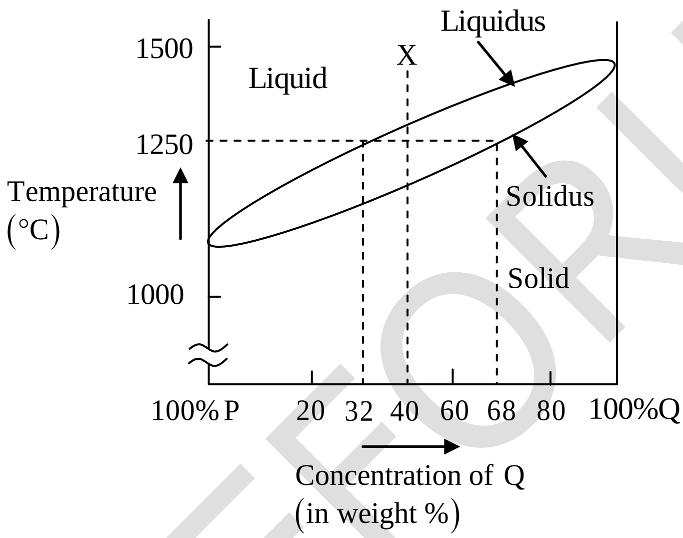
$$\left(\text{Drag force} \times \text{velocity}\right)_{\text{closed}} = \left(\text{Drag force} \times \text{velocity}\right)_{\text{open}}$$

$$= \left(\frac{1}{2}(C_d)_C \times eA \times V_C^2 \times V_C\right) = \left(\frac{1}{2}(C_d)_O \times eA \times V_O^2 \times V_O\right)$$

$$= 0.1 \times 44^3 = 0.8 \times V_0^3$$

$$V_0 = 22\,\text{km/hr}$$

49. The binary phase diagram of metals P and Q is shown in the figure. An alloy X containing 60% P and 40% Q (by weight) is cooled from liquid to solid state. The fractions of solid and liquid (in weight percent) at 1250°C, respectively, will be



(A) 22.2% and 77.8%

(B) 68.0% and 32.0%

(C) 32.0% and 68.0%

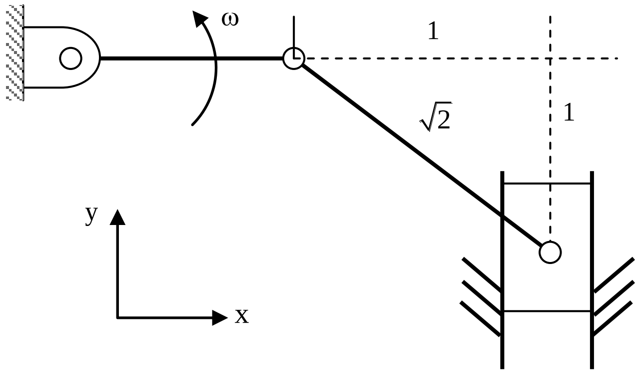
(D) 77.8% and 22.2%

Key: (A)

Sol: $m_s = \dfrac{c_o - c_\ell}{c_s - c_\ell} = \dfrac{40-32}{68-32} = 22.2\%$

$m_\ell = 77.8\%$

50. The crank of a slider-crank mechanism rotates counter clockwise (CCW) with a constant angular velocity ω, as sown. Assume the length of the crank to be r.

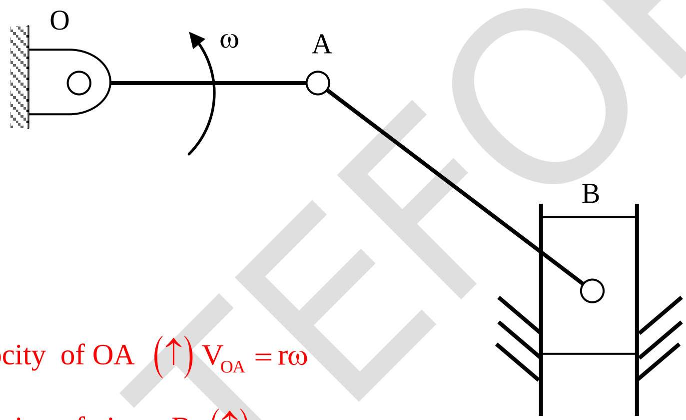


Using exact analysis. The acceleration of the slider in the y-direction, at the instant shown, where the crank is parallel to x-axis, is given by

(A) $-2\omega^2 r$ (B) $2\omega^2 r$ (C) $\omega^2 r$ (D) $-\omega^2 r$

Key: (C)

Sol:



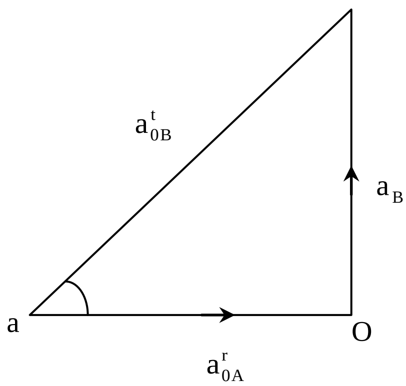
Velocity of OA $(\uparrow)$ $V_{OA} = r\omega$

Velocity of piston B $(\uparrow)$

Since both the vectors are parallel $V_{OA} = V_B$ and $V_{AB} = 0$

Radial acceleration of OA $a^r_{OA} = r\omega^2$ $(\leftarrow)$

Tangential acceleration of AB a^t_{AB}



$\tan 45° = \dfrac{a_B}{a^r_{OA}}$

$a_B = r\omega^2 \tan 45° = r\omega^2$

51. The probability that a part manufactured by a company will be defective is 0.05. If such parts are selected randomly and inspected, then the probability that at least two parts will be defective is ____ (round off to two decimal places).

Key: (0.17)

Sol: Given, the probability of manufactured part will be defective i.e,

$P(D) = 0.05 \Rightarrow q = 1 - 0.05 = 0.95$

Number of trials = n= 15 (15 parts are selected)

The R.V $X \rightarrow$ denote the number of defective parts

$\therefore P[X \geq 2] = ?$

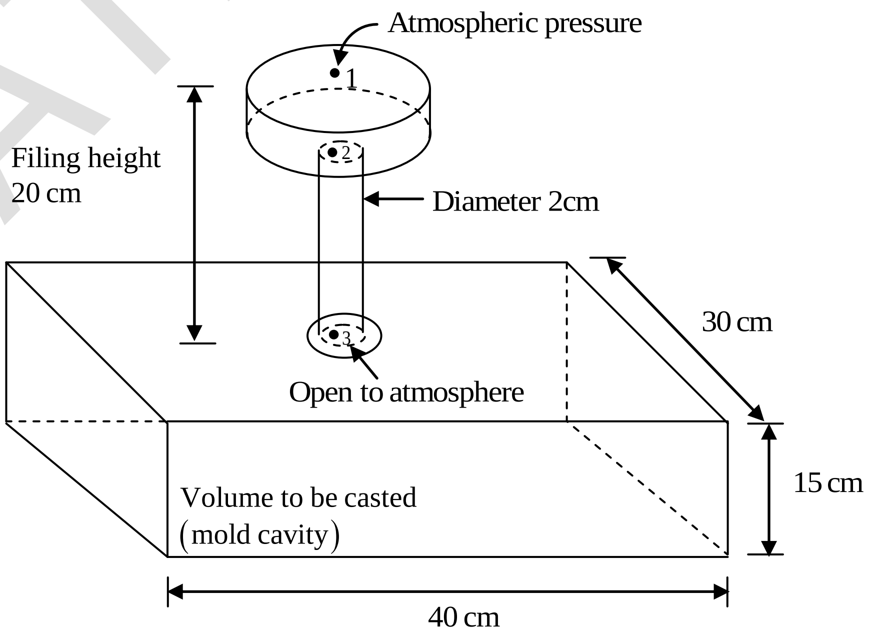
$\Rightarrow P[X \geq 2] = 1 - P(x < 2) = 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \{15_{C_0}.p^O q^{15} + 15_{C_1} p^1 q^{14}\}$, Using Binomial distribution

$= 1 - \{q^{15} + 15pq^{14}\} = 1 - \{(0.95)^{15} + 15(0.05)(0.95)^{14}\}$

$= 1 - \{0.46 + 0.37\} \approx 1 - 0.83 \approx 0.17$

52. The figure shows a pouring arrangement for casting of a metal block. Frictional losses are negligible. The acceleration due to gravity is 9.81 m/s^2. The time (in s, round off to two decimal places) to fill up the mold cavity $(\text{of size } 40\text{ cm} \times 30\text{ cm} \times 15\text{ cm})$ is_____



Key: (28.94)

Sol: $V_g = \sqrt{2gh_t} = \sqrt{2\times 9.81\times 0.2}$

$V_g = 1.98\,\text{m/s}$

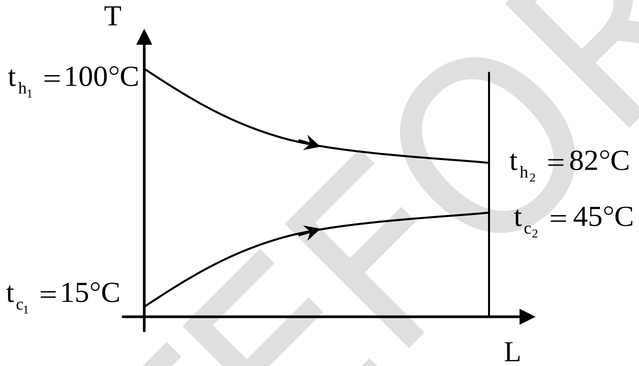
$$t_{f_1} = \frac{30\times 40\times 15}{\frac{\pi}{4}\times (2)^2\times 198} = \frac{1200\times 4\times 15}{\pi\times 4\times 198}$$

$t_{f_1} = 28.94\,\text{sec}.$

53. Hot and cold fluids enter a parallel flow double tube heat exchanger at 100 °C and 15 °C, respectively. The heat capacity rates of hot and cold fluids are $C_h = 200\,\text{W/k}$ and $C_c = 1200\,\text{W/K}$, respectively. If the outlet temperature of the cold fluid is 45°C, the log mean temperature difference (LMTD) of the heat exchanger is __________ K (round of to two decimal places).

Key: (57.71)

Sol:



$C_h = 2000\,\text{W/K}$

$C_c = 1200\,\text{W/K}$

Heat lost by hot fluid = heat gained by cold fluid

$C_h\left(t_{h_1} - t_{h_2}\right) = C_c\left(t_{c_2} - t_{c_1}\right)$

$2000\left(100 - t_{h_2}\right) = 1200(45-15) \Rightarrow t_{h_2} = 82°C$

where $\theta_1 = t_{h_1} - t_{c_1} = 100 - 15 = 85^\circ C$

$\theta_2 = t_{h_2} - t_{c_2} = 82 - 45 = 37^\circ C$

$$\text{LMTD}(\theta_m) = \frac{\theta_1 - \theta_2}{\ell n(\theta_1/\theta_2)}$$

$$= \frac{85-37}{\ell n(85/37)} = 57.71^\circ C$$

$\therefore \text{LMTD}(\theta_m) = 57.71\text{K}$

54. The thickness of a sheet is reduced by rolling (without any change in width) using 600 mm diameter rolls. Neglect elastic deflection of the rolls and assume that the coefficient of friction at the roll-workpiece interface is 0.05. The sheet enters the rotating rolls unaided. If the initial sheet thickness is 2 mm, the minimum possible final thickness that can be produced by this process in a single pass is _____ mm (round of to two decimal places).

Key: (1.25)

Sol: $D = 600\text{mm}$

$R = 300\text{mm};\ \mu = 0.05$

$h_i = 2\text{mm}$

$h_f = ?$

$h_i - h_f = \mu^2 R$

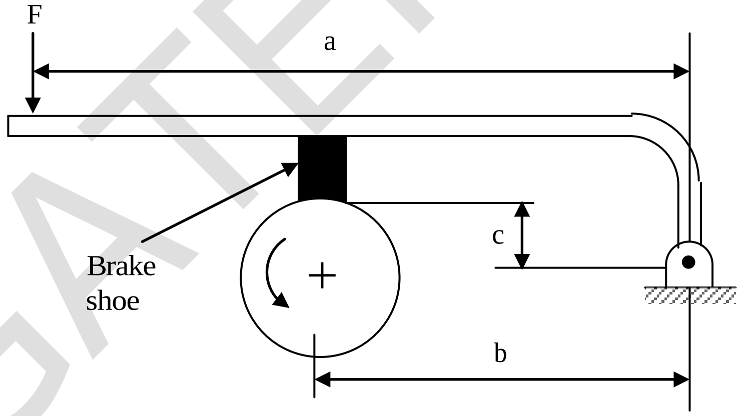
$h_i - h_f = 0.05^2 \times 300$

$h_i - h_f = 0.75$

$h_f = 2 - 0.75$

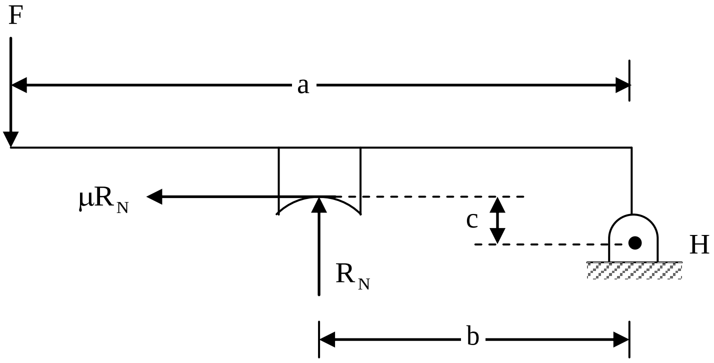
$h_f = 1.25\text{mm}$

55. A short shoe external drum brake is shown in the figure. The diameter of the brake drum is 500 mm. The dimensions a = 1000 mm, b = 500 mm and c = 200 mm. The coefficient of friction between the drum and the shoe is 0.35. The force applied on the lever F = 100 N as shown in the figure. The drum is rotating anti-clockwise. The braking torque on the drum is ____ N-m (round off to two decimal places).



Key: (20.34)

Sol:



$D = 500mm, \quad a = 1000mm, \quad b = 500mm$

$C = 200mm, \quad \mu = 0.35, \quad F = 100N$

$\Sigma M_H = 0$

$(F \times a) + (\mu R_N \times c) - (R_N \times b) = 0$

$(100 \times 1000) + (0.35 \times R_N \times 200) - R_N(500) = 0$

$R_N = 232.55N$

Braking torque $= T_B = \mu.R_N.R$

$$= 0.35 \times 232.55 \times 55 \times \frac{500}{2} \times 10^{-3} = 20.34N$$